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CONSTRUCTION AND ANALYSIS OF AN ANALYTICAL SOLUTION FOR THE SURFACE RAYLEIGH WAVE WITHIN THE FRAMEWORK OF THE COSSERAT CONTINUUM

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The problem of propagation of an acoustic surface Rayleigh wave in an infinite half-space is considered within the framework of the asymmetric theory of elasticity (Cosserat medium). It is assumed that material deformation is described not only by the displacement vector but also by an independent rotation vector. A global analytical solution of the problem in displacements is obtained. A comparative analysis of the solution obtained and the corresponding solution for the classical elastic medium is performed. Macroparameters characterizing the difference of the stress–strain state from that predicted by the classical theory of elasticity are introduced.

Key words: *Rayleigh wave, dispersion, Cosserat medium, analytical solutions.*

Introduction. The model of the medium whose deformation is described not only by the displacement vector $\boldsymbol{u}$ but also by a kinematically independent rotation vector $\boldsymbol{\omega}$, which are functions of coordinates and time, has riveted attention of researchers for a long time. This theory was called the moment or asymmetric theory of elasticity. The deformation behavior of elastic bodies in this theory has some specific features, namely, the elastic body, beginning from a certain characteristic scale and (or) at high gradients of stresses or strains, can acquire a stress–strain state significantly different from that predicted by the classical (symmetric) theory of elasticity.

In the theory of the Cosserat medium [1, 2], the vector $\boldsymbol{\omega}$ characterizes small turns of particles, and the tensors of stresses $\tilde{\sigma}$ and moment stresses $\tilde{\mu}$ are asymmetric. The dynamic behavior of an elastic isotropic medium with no allowance given to temperature effects is characterized by eight constants: two Lamé constants, four elastic constants characterizing the microstructure, density, and a parameter responsible for the measure of inertia of the medium during its rotation (density of the moment of inertia).

In many publications, the analysis of the moment behavior of the material is considered within the framework of a simplified medium called the medium with constrained rotation or the Cosserat pseudomedium [3]. Simplification is reached by using the dependence $\boldsymbol{\omega} = (1/2)\,\mathrm{rot}\,\boldsymbol{u}$, which, in particular, allows one to reduce the number of physical parameters from eight to five. Because of the drawbacks considered in detail in [1], however, the Cosserat pseudomedium model will not be further used in the present work.

A large amount of exact analytical solutions have been obtained within the framework of the asymmetric theory of elasticity (especially in the case of constrained rotation). In many papers, these solutions are analyzed and compared with the corresponding solutions of the classical theory of elasticity. In such a comparison, new physical constants determining the contribution of the moment components are normally chosen from the range of their energetically admissible values. This is caused by the lack of information on material constants with a microstructure, which is one of the main factors preventing the development of models of asymmetric media.

There are several papers where the physical constants of the Cosserat medium are determined. Thus, elasticity constants were measured in static experiments in [4]. Results of dynamic (in particular, ultrasonic)

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experiments were used for identification of the Leroux model and Cosserat pseudomedium [5] and for identification of the linear Cosserat medium [6, 7].

A number of effects observed, in particular, the dispersion of elastic surface Rayleigh waves [5] cannot be explained by the classical model of a continuous medium. This effect, however, finds an explanation within the framework of the Cosserat medium. It should be noted that the degree of decay of the Rayleigh wave amplitude with depth and wave ellipticity depend on the physical constants of the material, including parameters that describe the moment properties. This allows us to hope that this type of waves can be effectively used in possible experimental studies aimed at discovering the "moment" behavior of the material and at further determining the material parameters.

The problem of propagation of surface waves in a half-space was considered within the framework of the classical elastic medium, e.g., in [8, 9], in the Cosserat pseudomedium in [10], and in a micropolar medium in [11]. An analog of the Rayleigh equation for the Cosserat medium was derived in [12], but it should be noted that this solution does not allow a comprehensive analysis of the structure and decay of the Rayleigh wave.

An analytical solution in displacements, which describes propagation of the Rayleigh wave in the half-space, is obtained in the present work within the framework of the Cosserat medium. Based on the solution obtained, a parametric analysis of the ellipticity coefficient and the wavenumber is performed. As both parameters are measurable, the solution obtained can serve as a theoretical basis for possible experiments on determining new material parameters of an elastic Cosserat medium.

1. Formulation of the Problem. Let us write the basic relations for an elastic Cosserat medium [1]:

— equations of motion

$$\nabla \cdot \tilde{\sigma} + \boldsymbol{X} = \rho \ddot{\boldsymbol{u}}, \qquad \tilde{\sigma}^{\mathrm{t}} : \tilde{\boldsymbol{E}} + \nabla \cdot \tilde{\mu} + \boldsymbol{Y} = j\ddot{\boldsymbol{\omega}}; \tag{1.1}$$

— geometric relations

$$\tilde{\gamma} = \nabla \boldsymbol{u} - \tilde{\boldsymbol{E}} \cdot \boldsymbol{\omega}, \quad \tilde{\chi} = \nabla \boldsymbol{\omega}; \tag{1.2}$$

— physical equations

$$\tilde{\sigma} = 2\mu\tilde{\gamma}^{(S)} + 2\alpha\tilde{\gamma}^{(A)} + \lambda I_1(\tilde{\gamma})\tilde{e}, \qquad \tilde{\mu} = 2\gamma\tilde{\chi}^{(S)} + 2\varepsilon\tilde{\chi}^{(A)} + \beta I_1(\tilde{\chi})\tilde{e}. \tag{1.3}$$

With allowance for relations (1.1)–(1.3), the equations of motion for the displacement vector $\boldsymbol{u}$ and rotation vector $\boldsymbol{\omega}$ have the form

$$\begin{gathered}(2\mu + \lambda)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{u} - (\mu + \alpha)\,\mathrm{rot}\,\mathrm{rot}\,\boldsymbol{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\omega} + \boldsymbol{X} = \rho\ddot{\boldsymbol{u}},\\ (\beta + 2\gamma)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\omega} - (\gamma + \varepsilon)\,\mathrm{rot}\,\mathrm{rot}\,\boldsymbol{\omega} + 2\alpha\,\mathrm{rot}\,\boldsymbol{u} - 4\alpha\boldsymbol{\omega} + \boldsymbol{Y} = j\ddot{\boldsymbol{\omega}}.\end{gathered} \tag{1.4}$$

In (1.1)–(1.4), $\boldsymbol{X}$ is the vector of mass forces, $\boldsymbol{Y}$ is the vector of mass moments, $\tilde{\gamma}$ is the strain tensor, $\tilde{\chi}$ is the bending-torsion tensor, $\tilde{\sigma}$ is the stress tensor, $\tilde{\mu}$ is the moment stress tensor, μ and λ are the Lamé constants, α, β, γ, and ε are physical constants of the material within the framework of the elastic Cosserat medium, ρ is the density, j is the density of the moment of inertia (measure of inertia of the medium during its rotation), $\tilde{\boldsymbol{E}}$ is the Levi-Civita tensor of the third rank, $(\,\cdot\,)^{(S)}$ is the symmetrization operation, $(\,\cdot\,)^{(A)}$ is the alternation operation, $\nabla(\,\cdot\,)$ is the nabla operator, $I_1(\cdot)$ is the first invariant of the tensor, and $\tilde{e}$ is the unit tensor [13]. In contrast to the classical theory, the tensors $\tilde{\gamma}$ and $\tilde{\sigma}$ are asymmetric.

Let us consider a half-space whose surface is free from loads if there are no mass forces and moments. The Cartesian coordinate axes x and y are directed over the surface, and the z axis is directed inward the half-space (Fig. 1).

The solution of system (1.4), which describes the surface wave, is sought in the form

$$\begin{gathered}\boldsymbol{u}(x,z,t) = \{U_x(z), 0, U_z(z)\}\,\mathrm{e}^{i(kx - ft)},\\ \boldsymbol{\omega}(x,z,t) = \{0, W_y(z), 0\}\,\mathrm{e}^{i(kx - ft)},\end{gathered} \tag{1.5}$$

where $i = \sqrt{-1}$, k is the wavenumber, f is the circular frequency, and $U_x(z)$, $U_z(z)$, and $W_y(z)$ are decay functions depending on frequency; the physical meaning can be found only in real parts of these complex-valued functions.

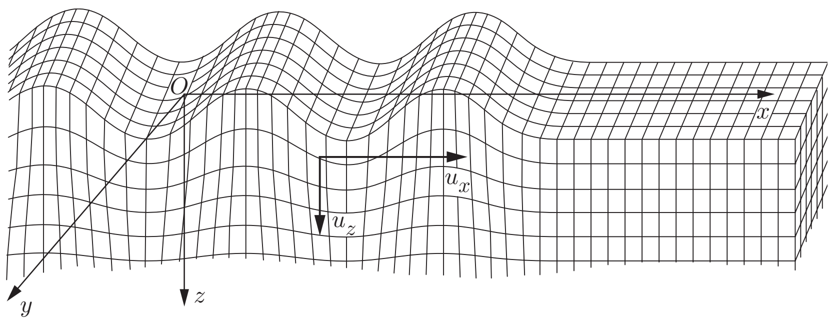

Fig. 1

The boundary conditions on the free surface of the half-space are set in the form

$$\sigma_{zx} = 0, \quad \sigma_{zy} = 0, \quad \sigma_{zz} = 0, \quad \mu_{zx} = 0, \quad \mu_{zy} = 0, \quad \mu_{zz} = 0. \tag{1.6}$$

2. Construction of the Solution. Substituting the displacement and rotation vector (1.5) into Eq. (1.4), we obtain a system of linear differential equations with respect to the functions $U_x(z)$, $U_z(z)$, and $W_y(z)$:

$$\begin{gathered}
(\alpha+\mu)\,\frac{d^2}{dz^2}\,U_x(z) + (\rho f^2 - k^2(\lambda+2\mu))U_x(z) + ik(\lambda+\mu-\alpha)\,\frac{d}{dz}\,U_z(z) - 2\alpha\,\frac{d}{dz}\,W_y(z) = 0,\\
(\lambda+2\mu)\,\frac{d^2}{dz^2}\,U_z(z) + (\rho f^2 - k^2(\alpha+\mu))U_z(z) + ik(\lambda+\mu-\alpha)\,\frac{d}{dz}\,U_x(z) + 2ik\alpha W_y(z) = 0,\\
(\gamma+\varepsilon)\,\frac{d^2}{dz^2}\,W_y(z) + (jf^2 - k^2(\gamma+\varepsilon) - 4\alpha)W_y(z) + 2\alpha\,\frac{d}{dz}\,U_x(z) - 2ik\alpha U_z(z) = 0.
\end{gathered} \tag{2.1}$$

For convenience of solution representation, we bring all quantities to a dimensionless form by using the characteristic size X_0 and frequency f_0. In addition, we introduce two dimensionless quantities, one of them depending on the characteristic size. We have already used these quantities previously to analyze analytical solutions of two-dimensional static problems for the Cosserat medium [14]:

$$A = X_0\sqrt{\frac{\mu}{B(\gamma+\varepsilon)}}, \qquad B = \frac{\alpha+\mu}{\alpha}. \tag{2.2}$$

To take into account the dynamic effects, we use four dimensionless parameters; two of them are analogs of velocities of the longitudinal and transverse waves, and the other two parameters are caused by the presence of new material constants of the Cosserat medium:

$$C_1^2 = \frac{\lambda+2\mu}{\rho X_0^2 f_0^2}, \quad C_2^2 = \frac{\mu}{\rho X_0^2 f_0^2}, \quad C_3^2 = \frac{\alpha+\mu}{\rho X_0^2 f_0^2}, \quad C_4^2 = \frac{\gamma+\varepsilon}{j X_0^2 f_0^2}. \tag{2.3}$$

Thus, with allowance for relations (2.2), (2.3), the dimensionless system (2.1) acquires the form

$$\begin{gathered}
\frac{d^2}{dz^2}\,U_x(z) + \Big(\frac{f^2}{C_3^2} - k^2\,\frac{C_1^2}{C_3^2}\Big)U_x(z) + ik\Big(\frac{C_1^2 - C_2^2}{C_3^2} - \frac{1}{B}\Big)\frac{d}{dz}\,U_z(z) - \frac{2}{B}\,\frac{d}{dz}\,W_y(z) = 0,\\
\frac{d^2}{dz^2}\,U_z(z) + \Big(\frac{f^2}{C_1^2} - k^2\,\frac{C_3^2}{C_1^2}\Big)U_z(z) + ik\Big(\frac{C_1^2 - C_2^2}{C_1^2} - \frac{C_3^2}{BC_1^2}\Big)\frac{d}{dz}\,U_x(z) + \frac{2ikC_3^2}{BC_1^2}\,W_y(z) = 0,\\
\frac{d^2}{dz^2}\,W_y(z) + \Big(\frac{f^2}{C_4^2} - k^2 - \frac{4A^2B}{B-1}\Big)W_y(z) + \frac{2A^2B}{B-1}\,\frac{d}{dz}\,U_x(z) - \frac{2ikA^2B}{B-1}\,U_z(z) = 0.
\end{gathered} \tag{2.4}$$

To find a solution of this system, we can use the substitution of variables:

$$U_x(z) = ik\Phi(z) - \frac{d}{dz}\,\Psi(z), \quad U_z(z) = \frac{d}{dz}\,\Phi(z) + ik\Psi(z), \quad W_y(z) = \Omega(z). \tag{2.5}$$

This substitution allows us to convert Eq. (2.4) to

$$\frac{d^2}{dz^2}\,\Phi(z)+\Big(\frac{f^2}{C_1^2}-k^2\Big)\Phi(z)=0,$$

$$\frac{d^2}{dz^2}\,\Psi(z)+\Big(\frac{f^2}{C_3^2}-k^2\Big)\Psi(z)+\frac{2}{B}\,\Omega(z)=0, \tag{2.6}$$

$$\frac{d^2}{dz^2}\,\Omega(z)+\Big(\frac{f^2}{C_4^2}-k^2-\frac{4A^2B}{B-1}\Big)\Omega(z)-\frac{2A^2B}{B-1}\,\frac{d^2}{dz^2}\,\Psi(z)+\frac{2k^2A^2B}{B-1}\,\Psi(z)=0.$$

The solutions of system (2.6) corresponding to a decrease in wave amplitude with depth have the form

$$\Phi(z)=D_0\,\mathrm{e}^{-\nu_0 z},\qquad \Psi(z)=D_1\,\mathrm{e}^{-\nu_1 z}+D_2\,\mathrm{e}^{-\nu_2 z},$$
$$\Omega(z)=(B/2)[D_1(k^2-\nu_1-f^2/C_3^2)\,\mathrm{e}^{-\nu_1 z}+D_2(k^2-\nu_2-f^2/C_3^2)\,\mathrm{e}^{-\nu_2 z}]. \tag{2.7}$$

Solutions (2.7) and relations (2.5) allow us to represent the amplitude functions $U_x(z)$, $U_z(z)$, and $W_y(z)$ in the form

$$U_x(z)=ikD_0\,\mathrm{e}^{-\nu_0 z}+D_1\nu_1\,\mathrm{e}^{-\nu_1 z}+D_2\nu_2\,\mathrm{e}^{-\nu_2 z},$$
$$U_z(z)=-D_0\nu_0\,\mathrm{e}^{-\nu_0 z}+ikD_1\,\mathrm{e}^{-\nu_1 z}+ikD_2\,\mathrm{e}^{-\nu_2 z}, \tag{2.8}$$
$$W_y(z)=(B/2)[D_1(k^2-\nu_1-f^2/C_3^2)\,\mathrm{e}^{-\nu_1 z}+D_2(k^2-\nu_2-f^2/C_3^2)\,\mathrm{e}^{-\nu_2 z}],$$

where the constants D_k are found from the boundary conditions, and the eigenvalues ν_k are determined by the expressions

$$\nu_0=\sqrt{k^2-f^2/C_1^2},\qquad \nu_1=\sqrt{k^2-a_1},\qquad \nu_2=\sqrt{k^2-a_2},$$

$$a_{1,2}=\frac{C_3^2+C_4^2}{2C_3^2C_4^2}\,f^2-2A^2\pm\sqrt{\frac{(C_3^2-C_4^2)^2}{4C_3^4C_4^4}\,f^4-\frac{2A^2(C_2^2C_3^2-2C_3^2C_4^2+C_2^2C_4^2)}{C_2^2C_3^2C_4^2}\,f^2+4A^4}.$$

As was shown in [12], in solving system (2.1), the values of a_k are also determined by the solutions of the algebraic equation $Xa^2-Ya-Z=0$, where

$$X=\frac{(\mu+\alpha)(\gamma+\varepsilon)}{2\alpha},\qquad Y=\frac{j(\mu+\alpha)+\rho(\gamma+\varepsilon)}{2\alpha}\,f^2-2\mu,\qquad Z=2\rho\Big(1-\frac{jf^2}{4\alpha}\Big)f^2.$$

By substituting solution (2.8) into the boundary conditions (1.6), we obtain a system of linear algebraic equations with respect to the constants D_k (the boundary conditions are preliminary normalized):

$$\begin{bmatrix} 2k^2-f^2/C_2^2 & -2ik\nu_1 & -2ik\nu_2 \\ 2ik\nu_0 & 2k^2-f^2/C_2^2 & 2k^2-f^2/C_2^2 \\ 0 & (a_1-f^2/C_3^2)\nu_1 & (a_2-f^2/C_3^2)\nu_2 \end{bmatrix}\begin{bmatrix} D_0 \\ D_1 \\ D_2 \end{bmatrix}=0.$$

The condition of the zero determinant of this system yields an analog of the classical Rayleigh equation [8, 9] with respect to the unknown wavenumber k

$$\sqrt{k^2-a_1}\,(a_1-f^2/C_3^2)\Big[(2k^2-f^2/C_2^2)^2-4k^2\sqrt{k^2-a_2}\,\sqrt{k^2-f^2/C_1^2}\,\Big]$$
$$-\sqrt{k^2-a_2}\,(a_2-f^2/C_3^2)\Big[(2k^2-f^2/C_2^2)^2-4k^2\sqrt{k^2-a_1}\,\sqrt{k^2-f^2/C_1^2}\,\Big]=0 \tag{2.9}$$

and the conditions of relations between the constants:

$$D_1=\frac{i(f^2-2k^2C_2^2)(f^2-a_2C_3^2)}{2kC_2^2C_3^2(a_1-a_2)\nu_1}\,D_0,\qquad D_2=-\frac{i(f^2-2k^2C_2^2)(f^2-a_1C_3^2)}{2kC_2^2C_3^2(a_1-a_2)\nu_2}\,D_0.$$

Returning to the components of the displacement and rotation vector (1.5), we finally obtain

$$u_x(x,z,t)=F_0\Big(k\,\mathrm{e}^{-\nu_0 z}+\frac{F_1}{k}\;\mathrm{e}^{-\nu_1 z}-\frac{F_2}{k}\;\mathrm{e}^{-\nu_2 z}\Big)\,\mathrm{e}^{i(kx-ft-\pi/2)},$$

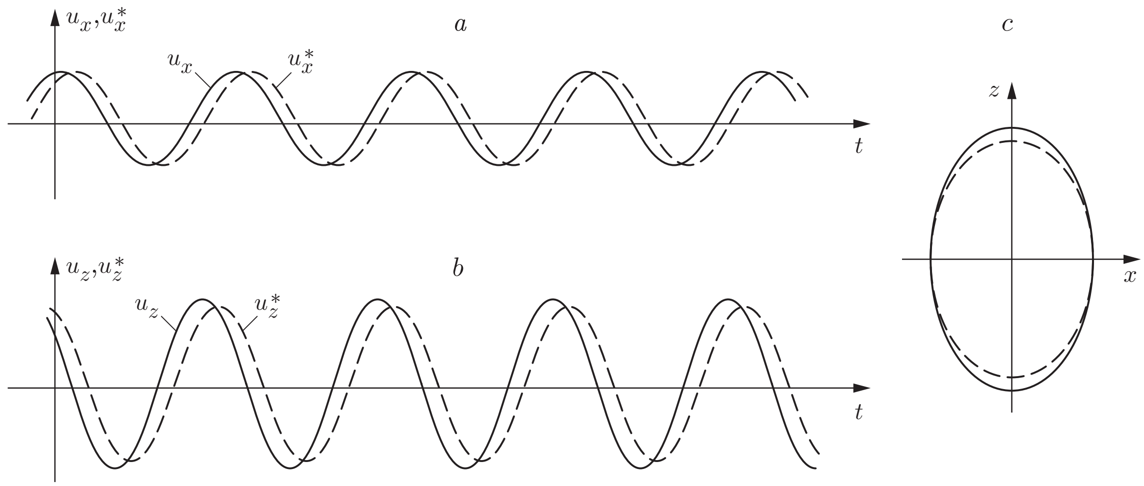

Fig. 2

$$u_z(x,z,t) = F_0\Big(\nu_0\, e^{-\nu_0 z} + \frac{F_1}{\nu_1}\, e^{-\nu_1 z} - \frac{F_2}{\nu_2}\, e^{-\nu_2 z}\Big)\, e^{i(kx-ft)}, \tag{2.10}$$

$$\omega_y(x,z,t) = F_0 B\Big[\frac{F_1}{2k\nu_1}\Big(a_1 - \frac{f^2}{C_3^2}\Big)\, e^{-\nu_1 z} - \frac{F_2}{2k\nu_2}\Big(a_2 - \frac{f^2}{C_3^2}\Big)\, e^{-\nu_2 z}\Big]\, e^{i(kx-ft-\pi/2)},$$

where F_0 is an undetermined constant, and the constants F_1 and F_2 are determined as

$$F_1 = \frac{(f^2 - 2k^2C_2^2)(f^2 - a_2C_3^2)}{2C_2^2C_3^2(a_1 - a_2)}, \qquad F_2 = \frac{(f^2 - 2k^2C_2^2)(f^2 - a_1C_3^2)}{2C_2^2C_3^2(a_1 - a_2)}. \tag{2.11}$$

Thus, relations (2.9)–(2.11) are the solution of the normalized system (1.4) with the boundary conditions (1.6).

3. Parametric Analysis of the Solution. The main reason for the parametric analysis is the search for qualitatively and quantitatively different quantities in the "moment" and classical cases. All quantities that refer to the classical medium will be further indicated by the asterisk.

For a numerical analysis, we used the values of the physical parameters cited in [6] and corresponding to human-bone parameters: $\lambda = 2.8 \cdot 10^{10}$ N/m^2, $\mu = 4 \cdot 10^9$ N/m^2, $\alpha = 4 \cdot 10^9$ N/m^2, $\gamma = 1.936 \cdot 10^8$ N, and $\varepsilon = 3.046 \cdot 10^9$ N. The value of the density of the moment of inertia j was not available in the literature, and we used an arbitrary value $j = 1 \cdot 10^{-3}$ kg/m^3.

Relation (2.7) shows that the component of the oscillatory process corresponding to the shear-wave potential Ψ exists simultaneously with the component characterized by the microrotation vector. These components of the wave process can exist separately only if $\alpha = 0$; in this case, Eqs. (2.6) describe the Rayleigh wave in the classical elastic half-space.

The form of solution (2.10) shows that the trajectory of motion of any particle is an ellipse whose major axis coincides with the z axis and whose minor axis coincides with the x axis. A typical form of the displacement components (2.10) on the half-space surface $z = 0$ for a certain distance Δx from the origin versus time is shown in Fig. 2a and b, and the form of the displacement component in the plane (x, z) is shown in Fig. 2c. The dashed curves refer to the solution for the classical medium [8] and solid curves refer to the new solution for the Cosserat medium.

The data plotted in Fig. 2 allow us to assume that, as the first experimentally measured quantity, we can choose the ellipticity coefficient $E = |u_x|/|u_z|$, which is independent of the coordinate x and time t on the half-space surface $z = 0$ but is a function of frequency (Fig. 3) and elastic parameters of the material. The influence of the moment description of the material behavior on the quantity δ_1 (presence of "moment" effects) becomes much more pronounced with increasing frequency. This is explained by the fact that the geometry and, hence, ellipticity of the Rayleigh wave for the classical elastic medium are independent of frequency.

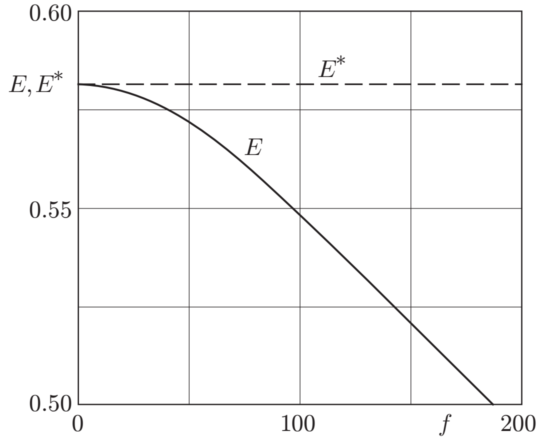

Fig. 3

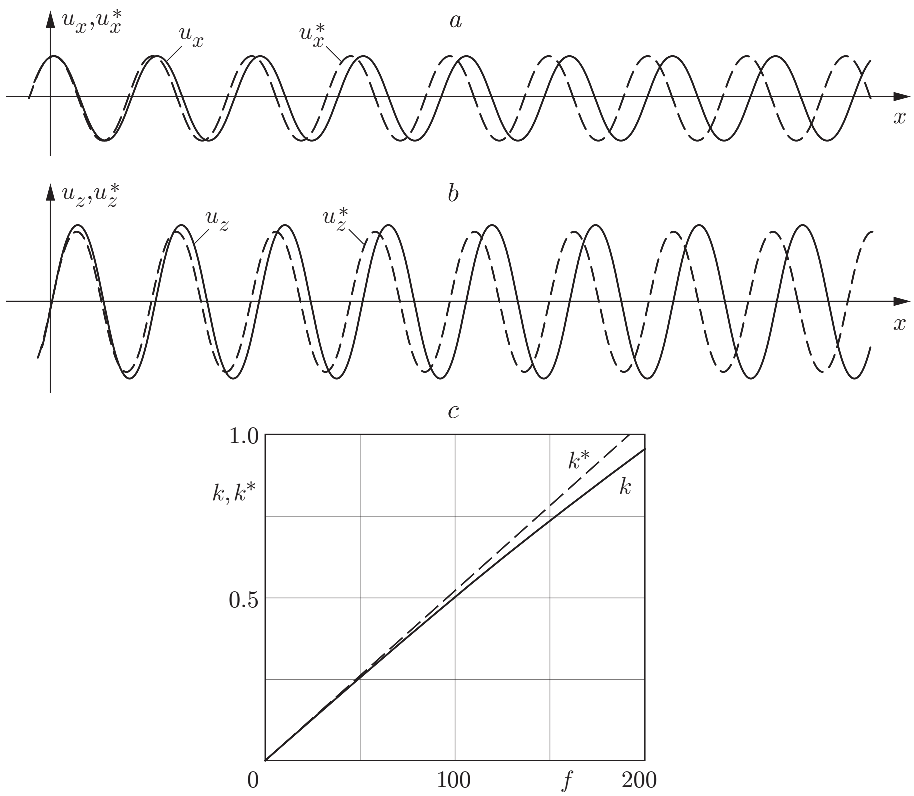

Fig. 4

Figure 4a and b shows the displacement-vector components as functions of the coordinate x for a certain time t_1 on the half-space surface. The solid and dashed curves correspond to the solutions for the "moment" and classical media, respectively. The wavenumbers of the classical and "moment" media are also different (Fig. 4c). Thus, on the basis of these dependences, the wavenumber measurement can be suggested as the second potential experiment.

The wavenumber cannot be measured directly, but we can use a simple computational procedure based on the use of the Fourier transform. For this purpose, we construct two signals $S_1(t) = u_z(x_1, 0, t)$ and $S_2(t) = u_z(x_2, 0, t)$ (x_1 and x_2 are fixed points at the x axis). The solid and dashed curves in Fig. 5a show the signals $S_1(t)$ and $S_2(t)$, respectively. We perform a continuous Fourier transform for both signals:

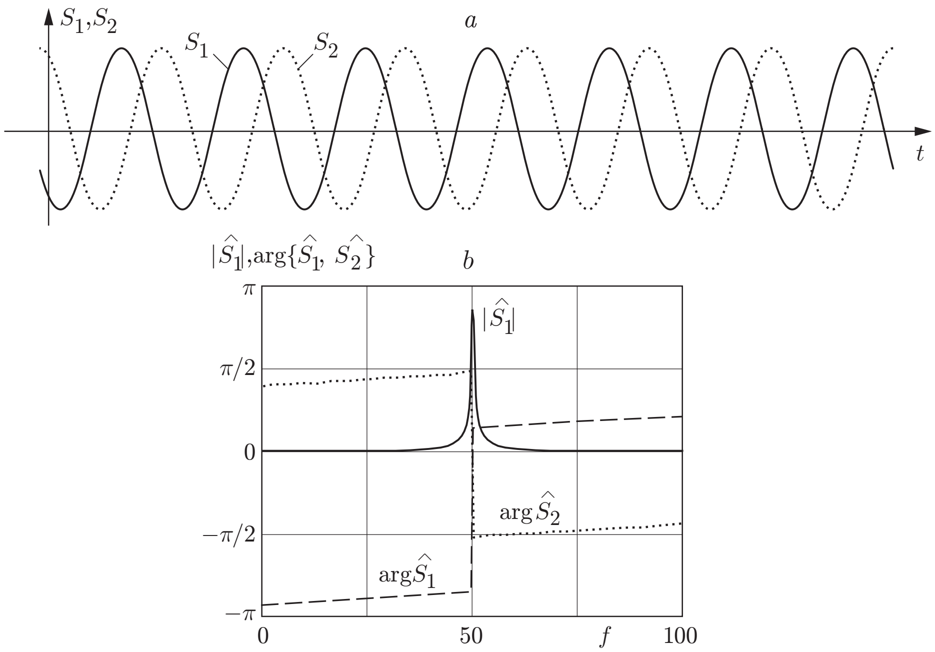

Fig. 5

$$\hat{S}_1(f) = \int_{-\infty}^{\infty} S_1(t)\,\mathrm{e}^{-ift}\,dt = F_0\Big(\nu_0 + \frac{F_1}{\nu_1} - \frac{F_2}{\nu_2}\Big)\,\mathrm{e}^{ik(f_s)x_1}\,\delta(f - f_s),$$

$$\hat{S}_2(f) = F_0\Big(\nu_0 + \frac{F_1}{\nu_1} - \frac{F_2}{\nu_2}\Big)\,\mathrm{e}^{ik(f_s)x_2}\,\delta(f - f_s).$$

Here $\hat{S}_1(f)$ and $\hat{S}_2(f)$ are complex functions of the Fourier image, f_s is the generating frequency of the signal, and $\delta(\,\cdot\,)$ is the Dirac delta function. Hence, the Fourier spectra of these two signals are related by the expression

$$\hat{S}_2(f) = \mathrm{e}^{ik(f_s)(x_2 - x_1)}\,\hat{S}_1(f), \tag{3.1}$$

which can be comparatively readily inverted by some optimization technique. Relation (3.1) is illustrated in Fig. 5b: the solid curve shows the absolute values of the Fourier coefficients of both signals, which coincide by virtue of the absence of dissipation, the dashed curve is the argument of the complex spectrum $\hat{S}_1(f)$, and the dotted curve is the argument of the spectrum $\hat{S}_2(f)$. Thus, relation (3.1) describes one possible way of experimental measurement of the wavenumber.

Finally, we also give the dependences characterizing the variation of velocity $C_r = f/k(f)$ and the decay of the Rayleigh wave. As is seen from Fig. 6a, the velocity in the Cosserat medium depends on frequency, i.e., in contrast to the classical medium, the Rayleigh waves in the Cosserat medium possess dispersion, which agrees with experimental investigations [7]. It is this fact, which have no analogs in the classical theory of elasticity, that can be used to verify the adequacy of the asymmetric theory of elasticity to real structural materials. The wave decay, i.e., the dependence of the displacement-vector components on the relative depth normalized to the wave length, is illustrated in Fig. 6b. This dependence shows that the Rayleigh wave is a typical surface wave, but its decay is little different from the classical case.

Conclusions. The qualitative and numerical analysis of the analytical solutions obtained in the present work and of the dependences plotted in Figs. 2–6 allows the following conclusions.

In contrast to two-dimensional static problems of the asymmetric theory of elasticity [14], the solution of the wave equation for the Rayleigh wave cannot be presented as the sum of classical and moment particular solutions.

In contrast to the classical case, the wave-ellipticity parameter depends on frequency. As the frequency increases, the difference between the values of this parameter for the classical and asymmetric cases becomes more

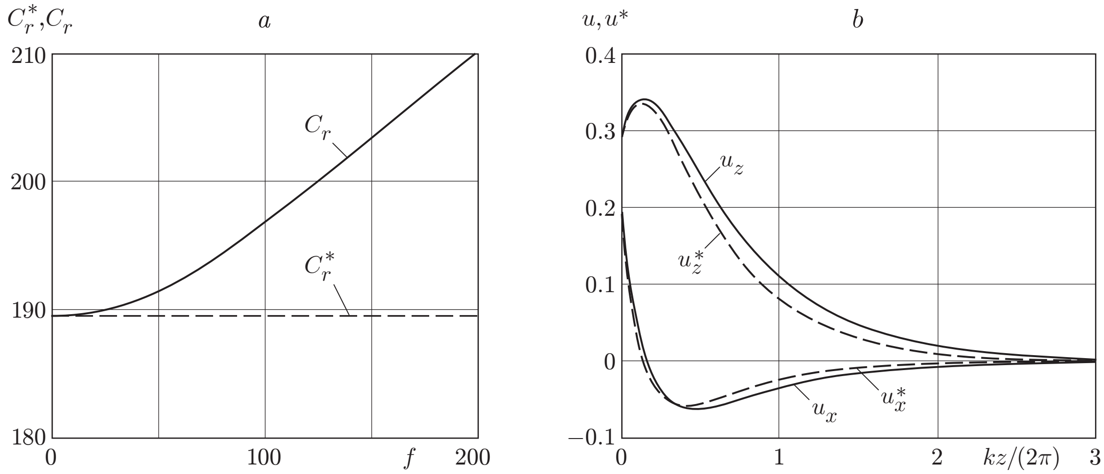

Fig. 6

noticeable. The difference between the wavenumbers for the classical and "moment" media also increases with increasing frequency. Both parameters can be measured in the course of an experiment aimed at discovering the effects of the "moment" properties of the medium. The information value of parameters from the viewpoint of manifestation of these properties is verified by the dependences presented.

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